FOUR YEAR UNDER GRADUATE (FYUG) PROGRAMME UNDER NEW EDUCATION POLICY, 2020

MATHEMATICS

Date of approval in Academic Council- 30th May 2024 and 21st June 2024.

Preface

The programme aims to lay a strong basic foundation for higher mathematics both in pure and applied branches of Mathematics. It is meant for students who wish to pursue their careers involving mathematical research and skills. The programme is intended to teach the students the art of problem solving activities in both the branches of Mathematics.

Programme Outcomes:

At the end of the course, students:

1. Will have a strong foundation in both the pure and applied Mathematics.
2. Will be able to ask logical questions and also be able to solve them.
3. Will be able to interact with people from outside the state and communicate their ideas effectively.
4. Will have a sound knowledge in programming and computation.

Structure of Mathematics Syllabus

1st Semester					
Course Code	**Title of the Course**	**Credit**			**Contact Hours**
		Theory	**Practical**	**Total**	
MTH-100	Fundamental Mathematics-I (Major)	4	-	4	60
MTH-100	Fundamental Mathematics-I (Minor)	4	-	4	60
MDC- 110….119 *MDC course offered by Mathematics is MDC-111	Any one of the available courses as notified by the University from time to time.	3	-	3	45
AEC- 120….129	Any one of the available courses as notified by the University from time to time.	3	-	3	45
SEC-130….. 139	Any one of the available courses as notified by the University from time to time.			3	45-90
VAC-140	Environmental Science	3	-	3	45
				20	
2nd Semester					
Course Code	**Title of the Course**	**Credit**			**Contact Hours**
		Theory	**Practical**	**Total**	
MTH-150	Fundamental Mathematics-II (Major)	4	-	4	60
MTH-150	Fundamental Mathematics-II (Minor)	4	-	4	60
MDC-160…..169	Any one of the available courses as notified by the University from time to time.	3	-	3	45
AEC- 170…..179	Any one of the available courses as notified by the University from time to time.	3	-	3	45
SEC-180……189	Any one of the available courses as notified by the University from time to time.			3	45-90
VAC-190…..199	Any one of the available courses as notified by the University from time to time.	3	-	3	45
				20	

3rd Semester					
Course Code	**Title of the Course**	**Credit**			**Contact Hours**
		Theory	**Practical**	**Total**	
MTH - 200	Calculus-I and Statics	4	-	4	60
MTH - 201	Group Theory	4	-	4	60
MDC-210... ...219	Any one of the available courses as notified by the University from time to time	3	-	3	45
AEC-220… …229	Any one of the available courses as notified by the University from time to time	2	-	2	30
SEC – 230	Analytical Thinking	3	-	3	45
VTC-240 … … 249	Any one of the available courses as notified by the University from time to time	1	3	4	105
				20	
4th Semester					
MTH - 250	Calculus-II	4	-	4	60
MTH - 251	Differential Equations	4	-	4	60
MTH - 252	Dynamics – I	4	-	4	60
MTH - 253	Matrix Theory and Vector Spaces	4	-	4	60
VTC-260… …269	Any one of the available courses as notified by the University from time to time	1	3	4	105
				20	
5th Semester					
MTH - 300	Calculus-III	4	-	4	60
MTH - 301	Number Theory and Ring Theory	4	-	4	60
MTH - 302	Numerical Methods and Optimization Techniques	4	-	4	60
MTH - 302	Elementary Algebra (Minor)	4	-	4	60
MTH-303	Internship	-	4	4	120
				20	
6th Semester					
MTH - 350	Complex Analysis	4	-	4	60
MTH - 351	Dynamics – II	4	-	4	60
MTH - 352	Discrete Mathematics	4	-	4	60
MTH - 352	Operations Research	4	-	4	60
VTC-360…369	Any one of the available courses as notified by the University from time to time.	1	3	4	105
				20	

Third Semester **Credits: 4**

MTH-200

Calculus-I and Statics (Major)
Total Marks: 100 (Internal: 25, External: 75)

Learning Objectives

The primary objective of this course is to introduce the concepts of sequence and series and their properties. The students will also learn some applications of differential calculus. From this course, the students will also understand necessary conditions for the equilibrium of particles acted upon by various forces and the concept of centre of gravity in some special cases.

Unit I: (Sequence and Series) (15 hours)

Sequences of real numbers: Definitions of bounded sequence, convergent sequence, limit of a sequence, monotonic sequence; examples; proof of the fact that monotonic and bounded sequences are convergent (using completeness of $\mathbb{R}$ as an axiom); Cauchy sequence; Cauchy's general principle of convergence; Infinite series of real numbers: partial sums, convergent series, comparison test, ratio test, Raabe's test, root test; absolute convergence; Leibnitz's theoremfor alternating series.

Unit II: (Applications of Calculus) (15 hours)

Tangent, normal, sub-tangent, subnormal, pedal equation. Asymptotes, radius of curvature, L'Hospital's rule, Taylor's and Maclaurin's theorem with Cauchy's form of remainders; Taylor's and Maclaurin's series; expansion of standard functions such as e^x, $\sin x$, $\cos x$, $\log(1 + x)$, $(1 + x)^n$.

Unit III: (Statics) (15 hours)

Composition and resolution of forces; parallelogram of forces, Components and resolved parts, Coplanar forces: Equilibrium of concurrent forces, Triangle of forces, Lami's Theorem and its converse. Parallel forces. Moment of a force; Definition, geometrical representation of Moments, Varignon's Theorem. Couples: definition, equilibrium of Couples, Equivalence oftwo Couples, Resultant of Couples, Resultant of a couple and a force.

Unit IV: (Statics) (15 hours)

Reduction of coplanar forces, Equilibrium of coplanar forces, Friction: laws of statical friction, laws of limiting friction, solution of problems on equilibrium of heavy bodies (such as uniformrods) resting on plane surfaces, Centre of gravity: CG of thin uniform rod, uniform lamina, triangular lamina and lamina in the form of a parallelogram and trapezium.

Course Outcome

This course will enable the students to:

- Recognize bounded, convergent, divergent, Cauchy and monotonic sequences and the limit of a bounded sequence. Apply various tests for convergence and absolute conver- gence of an infinite series of real numbers.

- Find tangent, normal, asymptotes, radius of curvature of some curves, the limits of indeterminate forms, series expansion of some standard functions.
- Find the components of a force, the resultant of a system of forces and couples.
- Determine the centre of gravity of some uniform bodies and analyze systems that include frictional forces.

Suggested Readings

(1) Statics (27th Edition), B. C. Das, B. N. Mukherjee, U. N. DHUR AND SONS PRIVATE LIMITED (2018).
(2) Differential Calculus (2nd Edition), R.K. Ghosh, K.C. Maity, New Central Book Agency Ltd. (2017).
(3) Introduction to Real Analysis (4th Edition), R.G. Bartle, D. R. Sherbert, Wiley India Edition (2021).
(4) Mathematical Analysis (6th Edition), S.C.Malik, S.C.Arora, New Age International Private Limited (2021).
(5) Thomas Calculus (14th Edition), J Hass, C Heil, M D Weir, Pearson Education (2018).
(6) Calculus: Early Transcendentals (7th Edition), James Stewart, Cengage India Private Limited (2017).
(7) Calculus (10th Edition), H. Anton, I. Bivens, S. Davis, Wiley, 10th Edition (2015).

Third Semester **Credits: 4**

MTH-201
Group Theory (Major)

Total Marks: 100 (Internal: 25, External: 75)

Learning Objectives

The learning objective of this course in group theory is to comprehensively grasp fundamental concepts, properties, and theorems of groups, enabling the understanding and application of abstract algebraic structures through various examples and problem-solving techniques.

Unit I: Foundations of Group Theory (15 hours)

Binary operations on a set, commutative and associative binary operations; identities and inverses, examples. Groups: definition, examples of groups such as $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$, $\mathbb{C}$, $\mathbb{Q}^*$, $\mathbb{R}^*$, $\mathbb{C}^*$, $\mathbb{Z}_n$, $M_2(\mathbb{R})$, General Linear group($GL_2(\mathbb{R})$), $\mathbb{R}^2$, $\mathbb{R}^3$, Special Linear Group (SL2($\mathbb{R}$)), nth roots of unity etc. laws of indices in both additive & multiplicative notation, right & left cancellation laws, uniqueness of identity and inverses, group tables of groups of low order (upto 8). Finite groups, order of a group and order of element of a group, examples of elements of finite and infinite order.

Unit II: Groups of Symmetries (15 hours)

Symmetries of a square, Dihedral Group, Quaternion Group. Permutation, Cyclic notation of permutation, permutation groups and properties, even and odd permutations, introductionand application of alternating groups.

Unit III: Subgroups (15 hours)

Subgroups, examples, elementary properties, cyclic subgroup generated by an element, center of a group, centralizer of an element, normalizer of an element. Cyclic group - generator of a group, order of an element in finite cyclic group, subgroups of cyclic groups, cosets and their properties, examples. Lagrange's Theorem and its application. Fermat's Little Theorem, Euler's Theorem and application.

Unit IV: Normal Subgroups (15 hours)

Normal subgroups, examples; conditions for a subgroup to be normal; examples; quotientgroup, homomorphisms, kernel and image of a homomorphism, isomorphism of groups-examples and elementary properties. Fundamental theorem of group homomorphism; isomorphism the- orems, automorphisms, inner automorphisms; examples.

Course Outcome

Upon completion of this group theory course, students will proficiently understand and apply fundamental concepts like binary operations, groups, and subgroups. They will demonstrate the ability to solve problems using theorems such as Lagrange's, Fermat's, and Euler's. Addi- tionally, students will identify and describe various group structures like cyclic and permutation groups, utilize mathematical reasoning to solve problems, grasp concepts of homomorphismsand isomorphisms, and apply group theory across different disciplines. This course aims to prepare students for advanced studies while fostering problem-solving skills and a solid under- standing of abstract algebraic structures.

Suggested Readings

(1) University Algebra, N.S. Gopala Krishnan, New Age International Pvt Ltd publishers (2021).
(2) Contemporary Abstract Algebra, J.A. Gallian, Narosa Book Distributory (2012).
(3) Topics in Algebra (Student Edition) I.N. Herstein, Wiley (2006).
(4) A First Course in Abstract Algebra, J.B. Fraleigh, Pearson Education India (2013).

Third Semester **Credits: 3**

SEC-230

Skill Enhancement Course
Analytical Thinking

Total Marks: 75 (Internal: 19, External: 56)

Learning Objectives

The objectives of this Course are:

- To introduce the basic concept of sets, relations, and mathematical logic.
- To develop critical and logical thinking in solving mathematical problems.

Unit I: Sets and Relations (15 hours)

Representation of sets - finite and infinite, subsets and equality of sets, power set, cardinality of a set. Venn diagrams, operation on sets - union, intersection, difference, symmetric difference, complement; Algebra of sets, ordered pair, cartesian product; Relations - symmetric, reflexive, transitive, antisymmetric, equivalence class, partition of sets.

Unit II: Combinatorics and Probability (15 hours)

Basic principle of counting, Factorial notation, Binomial theorem, permutation, permutations with repetitions, circular permutations; Combinations, combinations with repetitions, Restricted combination; Probability, Addition rule of probability, conditional probability, Multiplication rule of probability.

Unit III: Mathematical Logic (15 hours)

Introduction, propositions, truth table, negation, conjunction and disjunction. Implications, biconditional propositions, converse, contrapositive, and inverse propositions and precedence of logical operators. Propositional equivalence: Logical equivalences. Predicates and quantifiers: Introduction, Quantifiers, Binding variables and Negations.

Course Outcome

After the completion of this course, the learner will be able to:

- Apply concepts of sets, types of sets, and Venn diagrams
- Solve problems relating to probability and combinatorics
- Analyse a logical statement
- Differentiate between a logical statement and an ordinary statement

Suggested Readings

(1) Discrete Mathematics, S.K. Chakraborty, B.K. Sarkar, First Edition, Oxford University Press (2011).
(2) Numerical Analysis and Computational Procedures, S.A. Mollah, Eighth Edition, Books & Allied (P) Ltd (2022).
(3) Elements of Discrete Mathematics, M Chaudhary, V Sharma, P. Yadav, First Edition, Sultan Chand and Sons (2022).
(4) A Course on Mathematical Logic (2nd Edition), S.M. Srivastava, Springer (2013).
(5) Discrete Mathematics and Combinatorial Mathematics, R.P. Grimaldi, Fifth Edition, Pearson Education (2019).

Fourth Semester **Credits: 4**

MTH-250

Calculus II (Major)
Total Marks: 100 (Internal: 25, External: 75)

Learning Objectives

The primary goal of this course is to teach students how to integrate bounded functions on a closed and bounded interval, as well as how to extend it to cases where either the interval of integration is infinite or the integrand has infinite limits at a finite number of points on the interval of integration. The students will also learn of sequence and series of real-valued functions, as well as an essential class of function series. To comprehend the expansion of single variable integral calculus studies to functions with two or more independent variables.

Unit-I: Riemann Integration (15 Hours)

Riemann integral of functions of one variable; Darboux's theorem (statement and application); conditions for integrability; classes of bounded and integrable functions; properties of integrable functions; inequalities for integrals; functions defined by integrals; their continuity and differentiability; Mean value theorems for integrals.

Unit-II: Sequence and series of functions and Improper integral (15 Hours)

Pointwise and uniform convergence of sequence and series of functions, Weierstrass's M-test, Dirichlet test and Abel's test for uniform convergence, Uniform convergence and continuity, Uniform convergence and differentiability. Improper integrals; test for convergence when the integrand is non-negative; absolute convergence; tests for absolute and conditional convergence, beta and gamma functions; Abel's theorem, Dirichlet's theorem; Frullani's integral.

Unit-III: Double and Triple Integrals (15 Hours)

Line integrals in R^2, Double integration over rectangular and non-rectangular regions, Evaluation of Double Integrals, Triple integral over a parallelopiped and solid regions, Volume by triple integrals, Triple integration in cylindrical and spherical coordinates, Change of variables in double and triple integrals, Dirichlet integral.

Unit-IV: Green's, Stokes' and Gauss's Divergence Theorem (15 Hours)

Line integrals in R^3, Fundamental theorem for line integrals, Surface integrals, Green's theorem, Stoke's theorem, Gauss divergence theorem.

Course Outcome

The course will teach students how to:

(1) Learn some of the features of Riemann integrable functions and how to apply the fundamental integration theorems.
(2) Apply the various criteria for convergence of improper integrals.
(3) Investigate the interrelationships between the line integral, double integral, and triple integral formulations.
(4) Recognise the significance of Green's, Gauss', and Stokes' theorems in other areas of mathematics.

Suggested Readings

(1) A Course of Mathematical Analysis, S. Narayan, P.K.Mittal, S.Chand, Delhi (2007).
(2) Multivariable Calculus (7th Edition), J. Stewart, Brooks/Cole. Cengage (2012).
(3) Elementary Analysis: The Theory of Calculus (2nd Edition), K. A. Ross Springer (2013).
(4) Basic Multivariable Calculus, J. Marsden, A.J. Tromba & A. Weinstein, Springer India Pvt. Limited (2009).
(5) Mathematical Analysis, T.M. Apostol, Narosa Book Distributors (2002).

Fourth Semester **Credits: 4**

MTH-251

Differential Equations (Major)

Total Marks: 100 (Internal: 25, External: 75)

Learning Objectives

To impart the students a thorough knowledge of the fundamental concepts of differential equations and its classification. To explore the different techniques and methods of solving ordinary and partial differential equations. This will enhance the critical thinking skills and the ability of students to analyze and interpret differential equations arising in the real world.

Contents

Unit 1: Differential Equations of First Order (15 hours). First order first-degree differ- ential equations: separation of variables, substitution method, homogeneous differential equa- tions, exact differential equations, linear differential equations, Bernoulli's differential equations. First-order higher-degree differential equations: Equations solvable for p, equations solvable for x, equations solvable for y, Clairaut's equations.

Unit 2: Higher Order Differential Equations with Constant Coefficients (15 hours). Linear equations of second and third order with constant coefficients: Complementary Func- tions, Particular Integrals for x^m ($m \in \mathbb{N}$), e^{ax} $V \in$(V is any function of x), e^{ax}, sin(ax) or cos(ax), $x^m e^{ax}$, $e^{ax}\sin(mx)$, $e^{ax}\cos(mx)$, $x^n \sin(mx)$, $x^n \cos(mx)$.

Unit 3: Higher Order Differential Equations with Variable Coefficients (15 hours). Equations of type $a_1x^2y'' + a_2xy' + a_3y = f(x)$, Linear differential equations of second order with variable coefficients - homogeneous equations, exact equations, Normal form, transformation of the equation by changing the independent variable, method of variation of parameters, simultaneous equations of the form $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$, total differential equation $P\,dx + Q\,dy + R\,dz = 0$.

Unit 4: Partial Differential Equations (15 hours). Partial differential equations: Forma- tion of equation - by elimination of arbitrary constant, by eliminating the arbitrary function, solutions of linear equations of the first order - Lagrange's methods, Integral surfaces passing through a given curve, Orthogonal surfaces, Non-linear partial differential equations of the first order - Standard forms I, II, III, and IV, Charpit's method.

Course Outcome

Upon successful completion of this course, the students will be able to solve a variety offirst-order and higher-order linear and non-linear differential equations using various analytical techniques, and to understand the basics of partial differential equations, classifying and solving them. This will lay a solid foundation for further exploration of differential equations and its related disciplines in more advanced level courses.

Suggested Readings

(1) Ordinary and Partial Differential Equations, M.D. Raisinghania, S.Chand and Co. (2013).
(2) Integral Calculus (12th Edition), R.K. Ghosh, K.C. Maity, New Central Book Agency Ltd. (2013).

Fourth Semester **Credits: 4**

MTH-252

Dynamics - I (Major)

Total Marks: 100 (Internal: 25, External: 75)

Learning Objectives

(1) To understand how to apply Newton's second law of motion, differential equations, and initial conditions to analyze the motion of a particle under various forces.
(2) To analyze the motion of a particle in a plane.
(3) To understand how to use the work-energy theorem, the conservation of energy princi- ple, the impulse-momentum theorem, and the conservation of momentum principle to analyze the motion of a particle under different types of forces.
(4) To understand how to use the concepts of normal reaction, tension, friction, and con- straint to analyze the motion of a particle along a curve.

Unit I: Rectilinear motion (15 hours)

Rectilinear motion under variable laws of forces: Simple Harmonic Motion. Force of repulsion varying as displacement, motion under inverse square law, motion of a particle being attracted towards the centre of the Earth, motion under other laws of forces. Rectilinear motion in resisting medium: Rectilinear motion in resisting media on a horizontal plane where resistance varies as (i) velocity (ii) square of velocity (iii) displacement. Vertical motion under gravity where resistance varies as (i) velocity (ii) square of velocity.

Unit II: Uniplanar Motion (15 hours)

Velocity and acceleration in Cartesian coordinates, Radial and transverse velocities and accel-erations, Tangential and normal velocities and accelerations. Uniplanar motion of a Projectile: time of flight, horizontal range, greatest height, position and velocity at any time, path of a projectile is a parabola, projectiles required to pass through given points.

Unit III: Work, Power, Energy and Impulse (15 hours)

Work done by a force, power, Energy: kinetic energy, potential energy, work energy equation, potential function, conservative forces, principle of conservation of energy, impulse of a force, impulsive forces, conservation of linear momentum, motion of a shot and a gun. Collision of elastic bodies: laws of elastic impact, coefficient of restitution, direct impact, loss of energy due to collision, impulsive action between colliding spheres, oblique impact of two spheres.

Unit IV: Constrained motion of a particle (15 hours)

Motion of a particle compelled to move on a smooth plane curve, motion of a heavy particle compelled to move on a smooth plane curve in a vertical plane, motion on the inside of asmooth vertical circle, motion on the outside of a smooth vertical circle, parabolic motion after leaving the circle, motion on a smooth vertical cycloid.

Course Outcome

To develop the skills to solve problems involving rectilinear motion under variable laws of forces, uniplanar motion, work, power, energy, and impulse, constrained motion of a particleand to appreciate the applications of this course in physics and engineering.

Suggested Readings

(1) Dynamics, P.K. Gupta and R. Juneja, Ramesh Book Depot (2003).
(2) A Text Book on Dynamics for B.A./B.Sc students, M. Ray, S. Chand Publication (2002).
(3) Text Book of Dynamics, K.K. Singh, PHI Learning Pvt Ltd (2011).
(4) An Elementary Treatise on the Dynamics of a particle and of rigid bodies, S.L. Loney, New Age International Pvt Ltd (2016).

Fourth Semester **Credits: 4**

MTH-253

Matrix Theory and Vector Spaces (Major)

Total Marks: 100 (Internal: 25, External: 75)

Learning Objectives

(1) To study the basic properties of divisibility in number theory and to utilize the techniques of congruences in various ways for solving problems.
(2) To understand the properties of matrices and their applications in solving systems of equations.

Unit-I: Matrices and Elementary Operations (15 hours)

Matrices over $\mathbb{R}/\mathbb{C}$: Transpose, conjugate transpose, inverse of a matrix; diagonal, scalar, triangular matrices; similar matrices; nilpotent, idempotent, symmetric, skew-symmetric, hermitian, skew-hermitian matrices; trace of a square matrix; row rank/column rank of a matrix (in terms of linear independence of row/column vectors of a matrix); theorem on equality of row rank and column rank of a matrix; elementary operations, row/column reduced echelon form of a matrix, rank of a matrix, elementary matrices, determination of rank by elementary operations, determination of inverse of a matrix by elementary operations.

Unit-II: Matrices and Solutions of Equations (15 hours)

Adjoint of a matrix, inverse in terms of adjoints; determinantal rank of a matrix; equality of rank and determinantal rank; systems of linear equations, homogeneous and non-homogeneous equations, consistency and non-consistent system of equations, solving systems of equations by determining rank of augmented matrix and rank of coefficient matrix, Cramer's rule.

Unit-III: Vector Spaces and Linear Algebra (Contact hrs: 15)

Vector spaces, subspaces, and algebra of subspaces; quotient spaces, linear combination of vectors, linear span; linear independence, basis, and dimension.

Unit-IV: Linear Transformations (Contact hrs: 15)

Linear transformations, range space, null space, rank, nullity, and rank-nullity theorem; matrix representation of linear transformations; algebra of linear transformations, isomorphisms, and invertibility; eigenvalues and eigenvectors, characteristic polynomial; determinant, state- ment of Cayley-Hamilton Theorem.

Course Outcome

This course will provide the students a better understanding of matrices, and their applications to solve equations. They will be able to navigate vector spaces intricately, mastering linear transformations, eigenvalues, and eigenvectors. This will equip them with robust skills in linear algebra, bridging theoretical knowledge with practical problem-solving.

Suggested Readings

(1) Linear Algebra (2nd Edition), K. Hoffman, R. A. Kunze, PHI (1971)
(2) Linear Algebra (4th Edition), S H. Friedberg, A J. Insel, L.E. Spence,PHI (2004).
(3) Linear Algebra: A Geometric Approach, S. Kumaresan, PHI, (1999(.
(4) First Course in Linear Algebra, P.B. Bhattacharya, S.K. Jain, S.R. Nagpaul, New Age International (2015).

Fifth Semester **Credits: 4**

MTH-300

Calculus III (Major)

Total Marks: 100 (Internal: 25, External: 75)

Learning Objective

The objective of this course is to introduce functions of several variables to a student after he has taken a course in one variable calculus. The course will introduce partial derivativesand several of its consequences.

Unit I: Euclidean spaces (15 Hours)

Finite and Infinite Sets, Countable and Uncountable Sets, Basic properties of Euclidean distance function in R^n (n = 1, 2, 3); Neighbourhoods, Open sets, Closed sets, Limit points/ Accumulation points, Interior points in R^n (n = 1, 2, 3); Bolzano-Weierstrass theorem; Cantor intersection theorem (nested interval), Lindelof covering theorem, Compact sets; Heine-Borel theorem. Basic Ideas on Metric Spaces.

Unit II: Real valued function of several variables (15 Hours)

Limit, Continuity, Elementary properties of continuous functions; Continuity and Inverse images of open or closed sets, Continuous functions on compact sets; Special cases of continuous real-valued functions on closed, Bounded intervals of R: Bounds; Intermediate value theorem; Uniform continuity.

Unit III: Partial Differentiation (15 Hours)

Partial derivatives of a real-valued function with domain up to R^3; Existence of directional derivatives; Mean value theorem; Derivability of composites, Differentiability at a point, Condition for differentiability, Jacobians and their properties.

Unit IV: Differentiation and Extrema of Functions (15 Hours)

Reversal of order of derivatives, Schwarz's theorem, Young's theorem, Taylor's theorem in R^2, extreme values of a function of two variables, Necessary condition for extreme value, Sufficient condition for extreme value (for the function of two variables), Method of Lagrange multipliers.

Course Outcome

This course will enable the students to:

(1) Understand basic properties of open and closed sets, compact sets, Euclidean spaces and other related important theorems.
(2) Learn conceptual variations while advancing from one variable to several variables in calculus.
(3) Relate concepts of partial derivatives with directional derivatives and derivability of a function.
(4) Gain expertise in extreme values of functions for two variables.

Suggested Readings

(1) A Course of Mathematical Analysis, S. Narayan, S. Chand. Delhi, (2003).
(2) Mathematical Analysis, T.A. Apostol, Narosa Publishing House (2002).
(3) Mathematical Analysis-II, Sharma and Vasistha, Krishna Prakashan Publishers (2019).
(4) Introduction to Real Analysis, R.G. Bartle and D.R. Sherbert, John Wiley and Sons (2000).
(5) Introduction to Analysis, R.K. Ghosh and K.C. Maity, New Central Book Agency (2002).
(6) Advanced Calculus (2nd Edition), Widder and David, Dover Books on Mathematics (1989).

Fifth Semester **Credits: 4**

MTH-301

Number Theory and Ring Theory (Major)

Total Marks: 100 (Internal: 25, External: 75)

Learning Objectives

The principal objective of this course is to introduce Elementary Number Theory and some advanced topics in ring theory. The course is structured to provide students with some basic concepts of numbers and a comprehensive understanding of algebraic structures.

Unit 1: Divisibility and Congruences (15 hours)

Divisibility in the set of integers, basic properties, division algorithm, gcd, lcm and their properties, Euclidean algorithm, primes (in the set of natural numbers), fundamental theorem of arithmetic, Euclid's proof of the infinitude of primes, arbitrary gap in the distribution of primes. Congruences in the set of integers modulo a positive integer, basic properties, complete residue system, reduced residue system, Euler's phi function, Fermat's theorem, Euler's generalization of Fermat's theorem, applications, Wilson's theorem.

Unit 2: Congruences and Arithmetic Functions (15 hours)

Solution of congruences, linear congruences, Chinese remainder theorem, congruences of higher degree modulo a prime. Some functions of number theory: Greatest integer func- tion, elementary properties, Arithmetic functions, multiplicative functions, functions such as $\phi(n)$, $\mu(n)$, $\tau(n)$, σ, $\kappa(n)$.

Unit 3: Introduction to Rings (Contact hrs: 15)

Definition and examples of Rings; properties of Rings, Subrings, Integral domains, and Fields; Right, left and two-sided Ideals, Ideal generated by a subset, Operations on ideals, Prime and Maximal ideals; Finite integral domains, with $\mathbb{Z}_p$ as an example; Principal ideals; Examples of ideals in $\mathbb{Z}$, $\mathbb{Z}_n$, $M_n(R)$; Quotient ring, $\mathbb{Z}_n$ as a quotient ring.

Unit 4: Ring Homomorphisms and Integral Domains (Contact hrs: 15)

Ring homomorphisms, kernels, and properties; Isomorphism theorems including the cor- respondence theorem; Determination of ideals in $\mathbb{Z}_p$; Divisibility in integral domains; Units, associates, prime elements; Irreducible elements, GCD domain, Euclidean domain; Principal ideal domain, Unique factorization domains - definition, examples, and basic results.

Course Outcome

Upon successful completion of this course, students will be able to understand the con- cepts of divisibility, prime numbers, prime factorization theorem, congruences, and applying congruences in division processes. They will be able to comprehend the concepts engaged in ring theory and apply the Isomorphism theorems in understanding the structure of some basic examples of rings.

Suggested Readings

(1) An Introduction to the Theory of Numbers, I. Niven, H.S. Zuckerman and H.L. Montgomery, Wiley India Pvt. Ltd (2016).
(2) Elementary Number Theory (Seventh Edition), D.M. Burton, McGraw Hill Education (India) Private Limited (2012).
(3) Contemporary Abstract Algebra (4th Edition), J. A. Gallian, Narosa Publishing House (1999).
(4) Abstract Algebra (2nd Edition), M. Artin, Pearson (2011).
(5) A First Course in Abstract Algebra (7th Edition), J.B. Fraleigh, Pearson (2002).

Fifth Semester **Credits: 4**

MTH - 302

Numerical methods and Optimization Techniques (Major)

Total Marks: 100 (Internal: 25, External: 75)

Learning Objectives

The primary objective of this course is to empower the student with some optimization techniques upon formulating real-world problems into mathematical language.

Unit 1: Root finding, Linear Equations and interpolation (15 hours)

Bisection method and Newton-Raphson method; for solving polynomial equations; Gauss elimination method for solving system of equations.

Interpolation - Existence and uniqueness theorem for polynomial interpolation, Lagrange's interpolation polynomials; difference tables – divided difference, forward difference, backward difference; Newton's forward and backward interpolation formula;

Unit 2: Differentiation and Integration (15 hours)

Differentiation – first derivative using interpolation; Integration –trapezoidal rule, Simpson's 1/3-rule; Initial Value problems - Euler's method and Runge-Kutta method (Heun's method and Polygon method).

Unit 3: Solution of an LPP through Simplex Method (15 hours)

Slack and surplus variables, standard and canonical form of LPP, duality in LPP, basic solution of a system of simultaneous linear equations, basic feasible solution of an LPP, optimum basic solution of an LPP, Simplex algorithm - initial simplex tableau, pivot entry of a simplex tableau, optimality condition and condition for an unbounded solution, formation of a new tableau and terminal Simplex table.

Unit 4: Transportation Problem (15 hours)

Objective of the transportation problem, LPP formulation of the transportation problem, existence of feasible and optimal solutions, the transportation table, duality in the transportation problem, loops in transportation, solution of a transportation problem: obtaining an initial basic solution by NWC Rule, least-cost method or Vogel's approximation method, optimality test

and determining the net evaluations by the UV - method, degeneracy, transportation algorithm (MODI method).

Course Outcome

After this course, the student will be able to find the numerical solutions of algebraic/ transcendental equations, system of linear equations, differentiation and integration and to make calculated decisions upon encountering an optimization problems. They will also be able to apply numerical methods and optimization techniques not only in common day-to-day problems, but also to apply them in real life problems.

Suggested Readings

(1) Numerical Mathematical Analysis, J.B. Scarborough, Oxford and IBH Publishing Ltd (1930).
(2) Computer Programming in C, V. Rajaraman, PHI Private Limited, 2002.
(3) Numerical Methods, Problems and solutions, M.K. Jain, M. K. Iyenger, S.R.K. Jain, New Age International Publishers (2004).
(4) Numerical Methods, P. Kandasamy, K. Thilagavathy, K. Gunavathy, S. Chand & Co. (2006).
(5) Numerical Mathematics and computing, E.L. Cheney, D.K. Kincaid, Brooks/Cole Cengage Learning (2013).
(6) Operations Research, K. Swarup, P. K. Gupta and M.M. Singh, S. Chand (2002).
(7) Operations Research: An Introduction, P.K. Gupta and D. S. Hira, S. Chand (2002).
(8) Operations Research: Principles and Applications, G. Srinivasan, PHI (2010).
(9) Operations Research, Goel & Mittal, Pragati Prakashan (2014).

Fifth Semester **Credits: 4**

MTH-302

Elementary Algebra (Minor)

Total Marks: 100 (Internal: 25, External: 75)

Learning Objectives

The primary objective of this course is to introduce the students to basic algebraic concepts like sets, functions, matrices, and group theory.

Unit 1: Sets and Functions (15 hours)

Sets, subsets, and equality of sets; finite and infinite sets; relation on a set: reflexive, symmetric, anti-symmetric, transitive; examples from geometry and number systems; equivalence relation and equivalence classes. Functions/mappings, composition of maps, associativity, onto, one-one, bijective maps; inverse images of sets, inverse of a bijective map; proof of ”A function is invertible if and only if it is one-one and onto”; graph of a function: real-valued functions such as polynomials, rational functions, logarithmic functions, exponential functions.

Unit 2: Introduction to Matrices (15 hours)

A brief review of mxn matrix over R/C as a rectangular array of numbers (motivation through systems of linear equations); transpose, conjugate transpose; definition of inverse of a matrix; special types of matrices: diagonal, scalar, upper/lower triangular, nilpotent, idempotent, symmetric, skew-symmetric, Hermitian, skew Hermitian matrices.

Unit 3: Properties of Matrices (15 hours)

Trace of a square matrix; row vectors and column vectors of a matrix; row rank/column rank of an m x n matrix (in terms of linear independence of row/column vectors of the matrix);adjoint of a matrix; inverse in terms of adjoints; determinantal rank of matrix; equality of rank and determinantal rank; Elementary operations; elementary matrices; determination of the inverse of a matrix by elementary operations; rank of a matrix, determination of the rankby elementary operations; systems of linear equations: homogeneous and non-homogeneous.

Unit 4: Group Theory (15 hours)

Binary operations on a set, commutative and associative binary operations; identities and inverses, examples. Groups: definition, examples of groups such as $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$, $\mathbb{C}$, $\mathbb{Q}^*$, $\mathbb{R}^*$, $\mathbb{C}^*$, $\mathbb{Z}_n$, $M_2(\mathbb{R})$, General Linear group($GL_2(R)$), R^2, R^3, Special Linear Group(SL2(R)), nth rootsof unity etc. laws of indices in both additive & multiplicative notation, right & left cancellation laws, uniqueness of identity and inverses, group tables of groups of low order (upto 4). Finite groups, order of a group and order of element of a group, examples of elements of finite and infinite order.

Course Outcome

After this course, the student will be able to identify the different types of relations andfunctions and functions which are invertible. They will learn of the different properties of matrices and how to determine the rank and inverse of a matrix using various elementary operations. The students will also learn basic group theory concepts. Overall, the students will have a clear idea of basic algebraic concepts.

Suggested Readings

(1) University Algebra, N.S. Gopala Krishnan, New Age International Pvt Ltd publishers (2021).
(2) Contemporary Abstract Algebra, J.A. Gallian, Narosa Book Distributors (2012).
(3) A Course in Abstract Algebra, V.K. Khanna, S.K. Bhambri, Vikas Publishing House Pvt Ltd. (2017).
(4) Topics in Algebra (Student Edition), Wiley (2006).
(5) A First Course in Abstract Algebra, J.B. Fraleigh, Pearson Education India (2013).

Sixth Semester **Credits: 4**

MTH-350

Complex Analysis (Major)

Total Marks: 100 (Internal: 25, External: 75)

Learning Objectives

The primary objective of the course is to learn analysis of complex variables and functions. To study the geometry of the complex plane, complex sequences and series, analytic functions, and the techniques of differentiation and integration of complex-valued functions.

Unit I: Complex Number System (15 hours)

Complex numbers; field properties. Absolute value, conjugate: Basic properties; Real and imaginary parts; identification of the plane with the complex numbers; distance function (interms of the absolute value); triangle inequality, parallelogram law; related inequalities; polar representation; magnitude, argument; geometry of complex numbers: straight lines, circles in terms of complex numbers, equation of a circle through three points, equation of a half-plane; Spherical representation.

Unit II: Sequence, Series, Analytic Functions (15 hours)

Sequence of complex numbers; convergence, ratio test, root test, lim. Inf, lim. Sup, Cauchy sequence; Power series; absolute convergence, uniform convergence; Circle of convergence; Cauchy- Hadamard formula for the radius of convergence; ratio test; Analytic function; ba- sic properties; analyticity of power series; power series definition of (complex) exponential function, sine, cosine function; branch of logarithm; principal branch of a power function $z(\alpha)$ for a complex number α.

Unit III: Complex Integration (15 hours)

Differentiability, Cauchy Riemann theorem and its converse; integration of complex-valued function along a piece-wise differentiable curve (using real integral for real and imaginary parts); basic properties (including inequalities); Cauchy's theorem and its corollaries.

Unit IV: Cauchy's Integral Formula, Harmonic Functions (15 hours)

Cauchy's Integral formula for a disc, power series representation of analytic functions, Cauchy's estimate; Morera's theorem, Liouville's theorem; fundamental theorem of Algebra; Harmonic functions: basic properties, examples, and association with analytical functions.

Course Outcome

On accomplishing the course, the students will be equipped with the understanding of the fundamental concept of complex variable theory. The students will gain the skill to evaluate complex integral. The learners will be able to apply complex analysis techniques in various mathematical fields and also physics and engineering.

Suggested Readings

(1) Functions of one complex variable, J.B. Conway, Springer New York, (2012)
(2) Complex Analysis (2nd Edition), L.V. Ahlfors, McGraw-Hill Education (1990).
(3) Complex Variables and Applications, S. Ponnusamy and H. Silverman, Birhkäuser (2006).
(4) Complex Variables and Applications, R.V. Churchill, McGraw-Hill Education (1996).
(5) An Introduction to the Theory of functions of a complex Variable, E.T. Copson, Oxford University Press (1995).
(6) Visual complex Analysis (25th Anniversary Edition), T. Needham, OUP India (2023)

Sixth Semester **Credits: 4**

MTH-351
Dynamics - II (Major)

Total Marks: 100 (Internal: 25, External: 75)

Learning Objectives

(1) To study the behavior of objects under the influence of central forces, which act along the line joining the object and a fixed point. This is relevant in understanding the motion of planets, satellites, and other celestial bodies.
(2) Understanding the moment of inertia and product of inertia is crucial for analyzing the rotational motion of rigid bodies.
(3) To comprehend D'Alembert's Principle and its application in solving problems related to dynamic systems.
(4) Explore the motion of objects in two dimensions under the influence of finite forces.

Unit I: Central Forces (15 hours)

- Central forces, centre of force, central orbits, motion of a particle under a central force,use of reciprocal polar coordinates, equations of central orbits when law of force is given, to determine the law of force for a given central orbit, apse, apsidal distance, apsidal angle, stability of a nearly circular orbit with centre at the centre of force.
- Use of pedal coordinates and pedal equations, central force varying as the inverse square of the distance.

Unit II: Moments and Products of Inertia (15 hours)

- Moments and Products of Inertia, Some standard cases: uniform rod, a rectangular lamina, triangular lamina, circular ring, circular disc, rectangular parallelepiped, right circular cylinder, right circular cone. Theorems of parallel and perpendicular axes, Moment of Inertia about any line passing through the meeting point of three mutu- ally perpendicular axes, Moment and Product of inertia of a plane lamina, momental ellipsoid.

Unit III: Dynamics of rigid body (15 hours)

- D'Alembert's Principle, general equations of motion, motion of the centre of inertia, motion relative to the centre of inertia, centre of inertia in a straight line, impulsiveforces.
- Motion about a fixed axis: moment of the effective forces about the axis of rotation, expression for kinetic energy of the rigid body about the fixed axis, moment of momentum of the rigid body moving about the fixed axis.

Unit IV: Motion in two dimensions (Finite forces) (15 hours)

- Equation of motion of a rigid body in two dimensions, Kinetic energy of a body moving in two dimensions, moment of moment about the origin of a body moving in two dimensions, friction. Problems illustrating the above cases: motion of a rigid body down an inclined plane (sphere, cylinder, circular disc), slipping of rods, motion of a rod slidingdown in a vertical plane, motion on a horizontal plane (circular disc, sphere).

Course Outcome

Students who successfully complete a course in advanced dynamics will be prepared for further studies in physics, engineering, or applied mathematics.

Suggested Readings

(1) An elementary treatise on the dynamics of a particle and of rigid bodies, S.L. Loney, New Age International Pvt Ltd (2016).
(2) Dynamics of a particle, A.R. Vasistha, D.C. Agarwal, Krishna Prakashan Mandir Publication (2019).
(3) Rigid Dynamics, M.M. Rohman, New Central Book Agency (2018).
(4) Text Book of Dynamics (2nd Edition), F. Chorlton, CBS Publishers and Distributors (2019).

Sixth Semester **Credits: 4**

MTH - 352

Discrete Mathematics (Major)

Total Marks: 100 (Internal: 25, External: 75)

Learning Objectives

The core objective of the course is to learn a set of mathematical facts and apply them in problem solving. These mathematical themes include combinatorial analysis, lattices, Boolean structures, and discrete structures.

Unit I: Counting Techniques (15 hours)

Mathematical induction, principle of inclusion and exclusion, pigeonhole principle, finite combinatorics, generating functions, partitions, recurrence relations and recursive algorithms, linear recurrence relations with constant coefficients, homogeneous solutions, total solution, solution by the method of generating functions.

Unit II: Lattices (15 hours)

Definition, examples and basic properties of ordered sets, Hasse diagram, duality princi- ple, lattices as ordered sets, lattices as algebraic structures, sub lattices, direct product and homomorphisms, complete lattices.

Unit III: Boolean Algebra (15 hours)

Definition, examples and properties of modular and distributive lattices, Boolean algebras, Boolean polynomials, minimal forms of Boolean polynomials, Quinn-McCluskey method, Karnaugh diagrams, switching – circuits and applications of switching circuits.

Unit IV: Graphs (15 hours)

Definition, examples of graphs, simple graphs, degree of a vertex; handshaking lemma; paths, circuits, cycles, complete graphs, bipartite graphs, isomorphism of graphs, connected graphs, Eulerian paths, Euler's theorem, Hamiltonian paths, shortest path, adjacency and incidence matrices, cut points, bridges, trees and characterization of trees.

Course Outcome

The students will learn the counting principle and apply them in real-life problems. They will be able to apply the knowledge in analysis, design of electronic computers, dial telephones, switching systems, and many kinds of electronic control devices.

Suggested Readings

(1) Elements of Discrete Mathematics (2nd Edition), C.L. Liu, McGraw Hill Education (India) Pvt. Limited (2000).
(2) Introduction to Discrete Mathematics (4th edition), M.K. Sen and B.C. Chakraborty, Books and Allied Ltd. (2019).
(3) Introduction to Lattices and Order, B.A. Davey and H.A. Priestley, Cambridge University Press (2002).
(4) Discrete Mathematics with Graph Theory (3rd edition), E.G. Goodaire and M.M. Parmenter, Pearson Education Pvt Ltd, Indian Reprint (2006).
(5) A Textbook of Discrete Mathematics (9th edition), S.K. Sarkar, S.Chand (2016).
(6) A Textbook of Discrete Mathematics, H. Mittal and V.K. Goel, I.K. International (2010).

Sixth Semester **Credits: 4**

MTH-353

Operations Research (Major)

Total Marks: 100 (Internal: 25, External: 75)

Learning Objectives

The primary goal of this course is to upgrade the students' knowledge learnt in fifth semester with more Optimization techniques.

Unit I: Integer Programming (15 Hours)

Pure and mixed Integer Programming problems, Gomory's all - I.P.P. method, Construction of Gomory's constraints, Fractional Cut method - All Integer L.P.P. and Mixed Integer L.P.P., Branch and Bound method, Applications of Integer Programming.

Unit II: Assignment problem (15 Hours)

Mathematical formulation of assignment problem, the assignment method, special cases in assignment problems, a typical assignment problem, dual of the assignment problem, the travelling salesman problem.

Unit III: Markov Analysis (15 Hours)

Probability vector, stochastic matrix, regular stochastic matrix, fixed point of a square matrix, fixed probability vector of a square matrix, fixed probability vector of a regular stochastic matrix, Markov processes, state transition matrix, transition diagram, construction of a state transition matrix, n-step transition probabilities, initial probability distribution and n-step probability distribution, stationary distribution of a regular Markov chain, absorbing states.

Unit IV: Game Theory (15 Hours)

Definition of a game, two-person-zero–sum games, pure and mixed strategy, optimum strat- egy, value of the game, payoff matrix, the maximin–minimax principle, fair and strictly de- terminable game, saddle point, rule for determining a saddle point. Game without a saddle point, minimax and maximin criterion for a game without a saddle point, solution of a game, graphical solution of 2 n and m 2 games, dominance property and modified dominanceproperty, arithmetic method for solution of games up to order 4.

Course Outcomes

(1) After this course, the student will learn to solve problems on Integer Programming.
(2) They will also be able to make calculated decisions upon encountering assignment problems, use transition matrices in Markov chains and apply the techniques to Game Theory.

Suggested Readings

(1) Operarions Research, K. Swarup, P.K. Gupta and M.M. Singh, S. Chand (2002).
(2) Operations Research-An introduction, P.K. Gupta and Hira, S Chand (2002).
(3) Operations Research: An Introduction, H.A. Taha, Prentice Hall of India, (2006).
(4) Operations Researcg, Goel and 2Mittal, Pragati Prakashan, (2014).
(5) Theory and Problems of Finite Mathematics, S. Lipschutz, Schaums Outlines –McGraw Hill Book Company , (1983).